BENHA UNIVERSITY
FACULTY OF ENGINEERING (SHOUBRA)
ELECTRONICS AND COMMUNICATIONS ENGINEERING

CCE 304

Measurements and Instrumentations

(2022 - 2023) term 231

Lecture 1: Measurement Errors.

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Outlines

- Ch.1 : Measurement Systems, Units, and Standards.
- Ch.2 : Measurement Errors

Chapter 1:Measurement Systems, Units, and Standards

1.1 SI Mechanical units:

- **Fundamental Units:**

  Length (L): meter (m), Mass (M): kilogram (kg), Time (T): second (s)

- **Derived Units:**

  Area: Meter Squared

  Force: Newton (N) ⇒ **Force = mass × acceleration**

  Work: Joule (J) ⇒ **Work = force × distance**

  Power: Watt (W) ⇒ $power = \frac{work}{time}$

TABLE 1-2 SI Units, Symbols, and Dimensions

Quantity	Symbol	Unit	Unit symbol	Dimensions
Length	l	meter	m	$[L]$
Mass	m	kilogram	kg	$[M]$
Time	t	second	s	$[T]$
Area	A	square meter	m^2	$[L^2]$
Volume	V	cubic meter	m^3	$[L^3]$
Velocity	v	meter per second	m/s	$[LT^{-1}]$
Acceleration	a	meter per sec per sec	m/s^2	$[LT^{-2}]$
Force	F	newton	N	$[MLT^{-2}]$
Pressure	p	newton per square meter	N/m^2	$[ML^{-1}T^{-2}]$
Work	W	joule	J	$[ML^2T^{-2}]$
Power	P	watt	W	$[ML^2T^{-3}]$

1.2 Scientific Notation and Metric Prefixes:

- When working in electronics it is common to encounter very small and very large numbers.
- **Scientific Notation** is a means of using single-digit numbers plus powers of ten to express very large and very small numbers.

$$10\,000 = 1 \times 10 \times 10 \times 10 \times 10 = 1 \times 10^4 \qquad 0.015 = 1.5 \times 10^{-2}$$

- **Metric Prefix** : a letter symbols for the various multiples and submultiples of 10.

TABLE 1-1 Scientific Notation and Metric Prefixes

Value	Scientific notation	Prefix	Symbol
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	**giga**	**G**
1 000 000	10^{6}	**mega**	**M**
1000	10^{3}	**kilo**	**K**
100	10^{2}	hecto	h
10	10	deka	da
0.1	10^{-1}	deci	d
0.01	10^{-2}	centi	c
0.001	10^{-3}	**milli**	**m**
0.000 001	10^{-6}	**micro**	**µ**
0.000 000 001	10^{-9}	**nano**	**n**
0.000 000 000 001	10^{-12}	**pico**	**p**

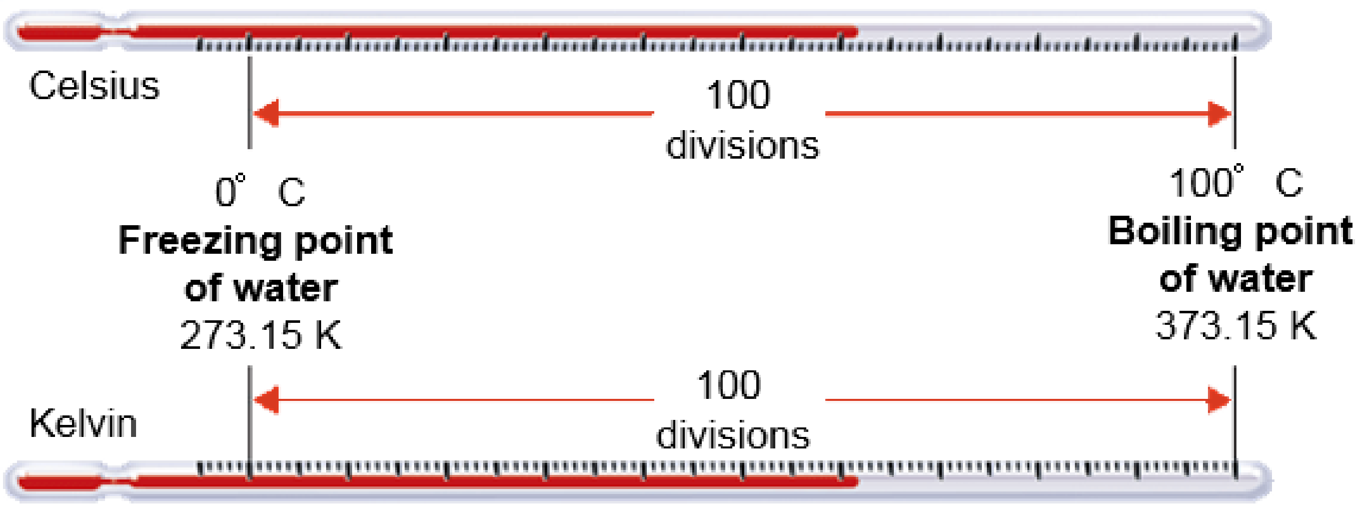
1.3 SI Electrical units:

- Electric current (I): A
- Electric charge (Q): C ⇨ Charge = current × time
- Voltage (V): V ⇨ $V = \frac{P}{I} = \frac{[ML^2T^{-3}]}{[I]}$
- Resistance (R): ohm ⇨ $R = \frac{V}{I} = \frac{[ML^2T^{-3}I^{-1}]}{[I]}$

Quantity	Symbol	Unit	Unit symbol	Dimensions
Electric current	I	ampere	A	$[I]$
Electric charge	Q	coulomb	C	$[IT]$
Emf	V	volt	V	$[ML^2T^{-3}I^{-1}]$
Electric field strength	ξ	volt per meter	V/m	$[MLT^{-3}I^{-1}]$
Resistance	R	ohm	Ω	$[ML^2T^{-3}I^{-2}]$
Capacitance	C	farad	F	$[M^{-1}L^{-2}T^4I^2]$
Inductance	L	henry	H	$[ML^2T^{-2}I^{-2}]$
Magnetic field strength	H	ampere per meter	A/m	$[IL^{-1}]$
Magnetic flux	Φ	weber	Wb	$[ML^2T^{-2}I^{-1}]$
Magnetic flux density	B	tesla	T	$[MT^{-2}I^{-1}]$

1.4 SI Temperature Scales:

- There are two temperature scales, the **Celsius** scale and the **Kelvin** (absolute) scale.



- **absolute zero:** the zero point on the Kelvin temperature scale, equivalent to –273.15°C
- In some countries as the United States, temperatures are usually given in **degrees Fahrenheit**.

$K = °C + 273$ $°C = K - 273$ $°C = \frac{°F - 32}{1.8}$

Outlines

- Ch.1 : Measurement Systems, Units, and Standards.
- Ch.2 : Measurement Errors

Chapter 2: Measurement Errors

1. Types of Measurement Errors.
2. Absolute and Relative Errors.
3. Measurements Characteristics.
4. Measurement Error Combinations.

Introduction:

- No electronic component or instrument **is perfectly accurate;** all have some error or inaccuracy.
- These errors are introduced due to either **defect** in the instrument, **wrong observance**, or **environmental factors**.
- These errors could combine to either:
  - Completely cancel each others.
  - Create greater errors in measurement (Worst case)
- The worst case should always consider while performing measurement, where these errors could combine to create larger error.

2.1 Measurement Errors types:

Measurement errors can be categorized into three types:

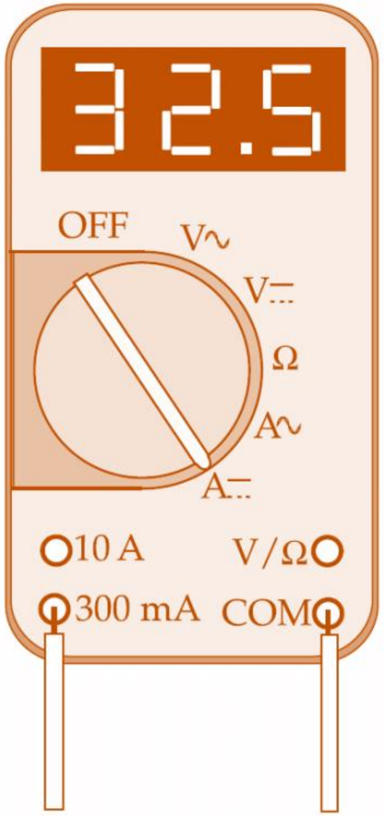
- Measurement Errors
  - Gross Error
  - Systematic Error
  - Random Error

Gross Error (Human Error)

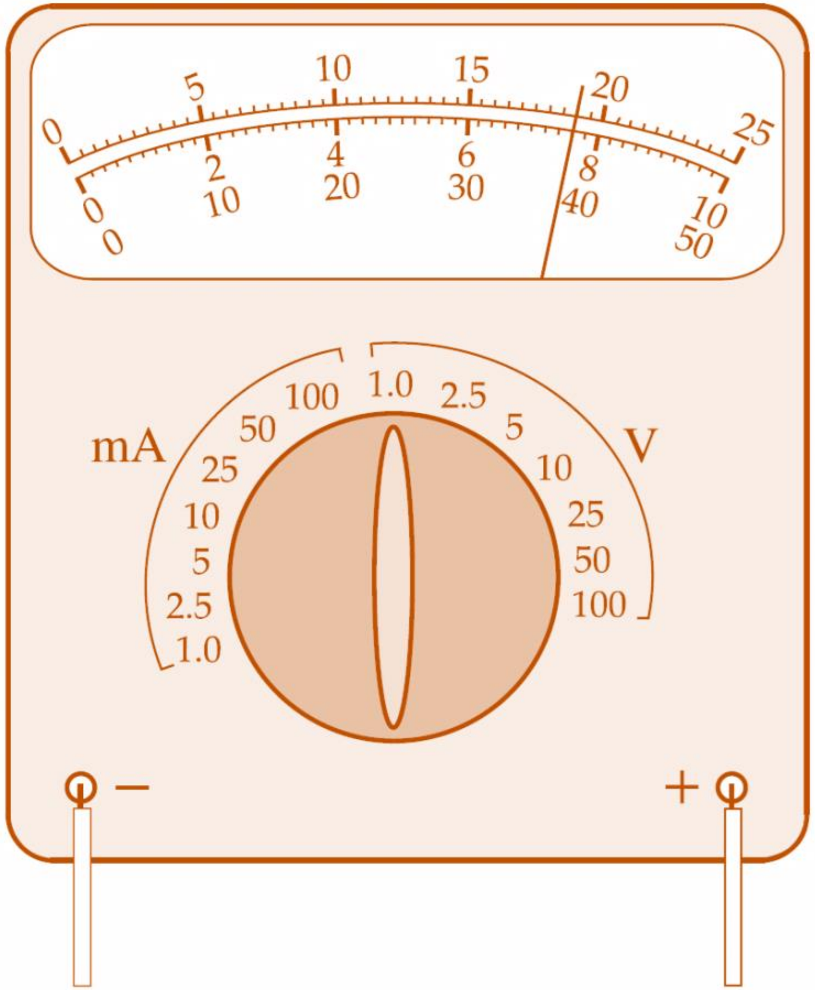
Errors due to **human mistakes** in using instruments, recording observations, and calculating measurement results.

Example

- Misunderstanding the unit in case of digital devices (21 V instead of 21 mV).
- A wrong scale may be chosen in analog instruments.
- Transpose of the readings while recording. (24.9 mV instead of 29.4 mV).
- Observational Errors: Errors introduced by the observer as the **parallax error**.



(a) Digital instrument indicating 32.5 mA

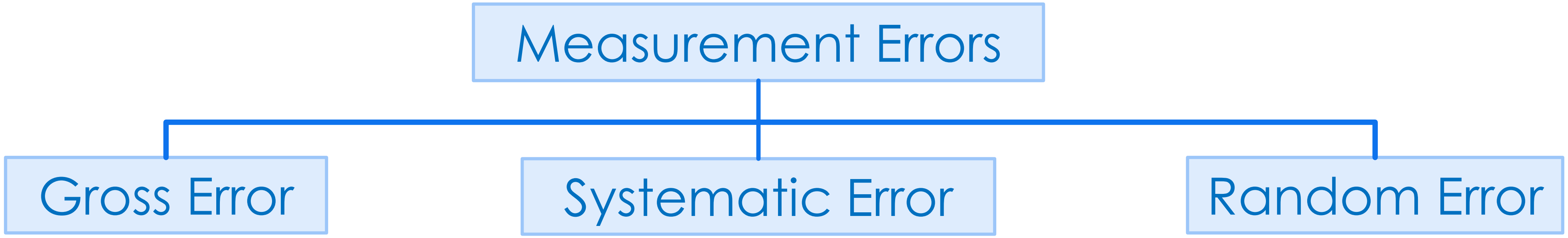


(b) Analog instrument indicating 0.76 V

Figure 2-1 Serious measurement errors can occur if an instrument is not read correctly. The digital instrument is on a 300 mA range, so its reading is in milliamperes. For the analog meter, the range selection must be noted, and the pointer position must be read from the correct scale.

2.1 Measurement Errors types:

Measurement errors can be categorized into three types:



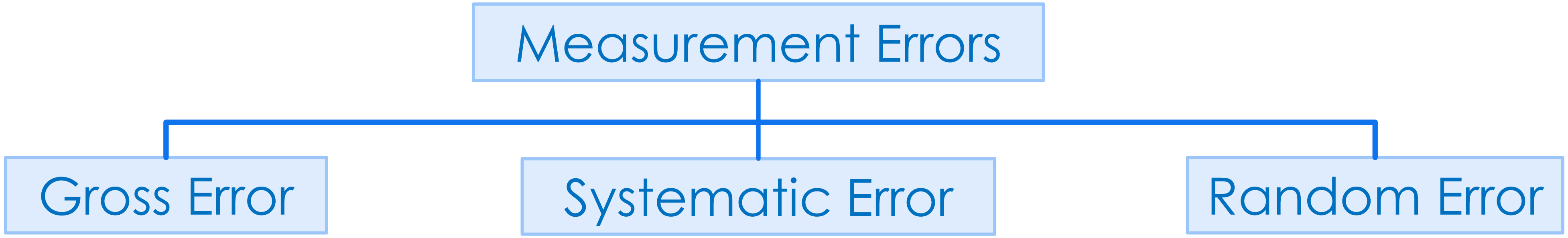
Systematic Error

Errors due to **problems with instruments.**

- **Instrument Errors:** May be due to incorrect device calibration.
- **Environmental Errors:** Change in environnemental conditions may change some of device parameters.

2.1 Measurement Errors types:

Measurement errors can be categorized into three types:



Random Error

Errors due to **unknown factors.**

- These errors are relatively **small.**
- These errors can be **reduced** by **increasing** the number of readings and using arithmetic mean.

Chapter 2: Measurement Errors

1. Types of Measurement Errors.
2. Absolute and Relative Errors.
3. Measurements Characteristics.
4. Measurement Error Combinations.

2.2 Absolute and Relative Errors:

- The error in measuring instruments can be represented in two ways: **Absolute** and **Relative**

Absolute Error (Δe)

It is defined as the difference between the measured A_m and the true A_t values.

$$\Delta e = A_m - A_t$$

Example

An ammeter reads 6.7 A and the true value of the current is 6.54 A. The absolute error is

$$\Delta e = A_m - A_t = 6.7 - 6.54 = 0.16\ A$$

2.2 Absolute and Relative Errors:

Relative Error (e_r)

It is defined as the ratio of the absolute error Δe to the true value A_t of the quantity being measured.

$$e_r = \frac{\Delta e}{A_t}$$

Percentage error

$$\%e_r = e_r \times 100 = \frac{\Delta e}{A_t} \times 100$$

Example

The current through a resistor is 2.5 A, but the measurement yields a value of 2.45 A.

The absolute error is

$$\Delta e = A_m - A_t = 2.45 - 2.5 = -0.05A$$

The relative error

$$e_r = \frac{\Delta e}{A_t} = \frac{-0.05}{2.5} = -0.02$$

The percentage relative error

$$\%e_r = e_r \times 100 = -2\%$$

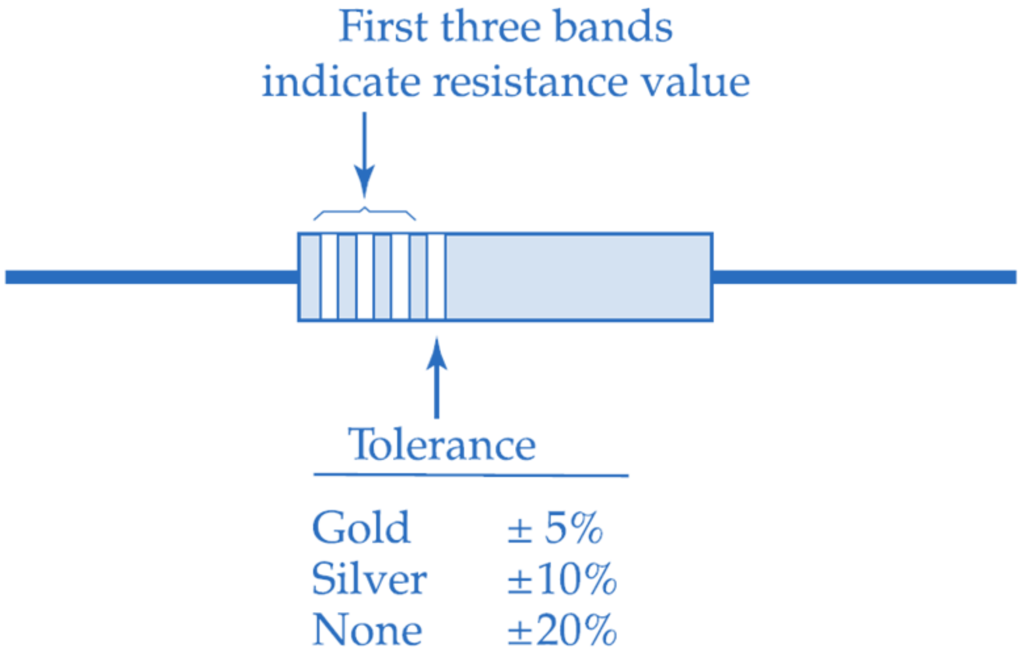


Figure 2-2 The relative error in a measured or specified quantity is expressed as a percentage of the quantity. The absolute error is determined by converting the relative error into an absolute quantity.

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2.3 Measurements Characteristics:

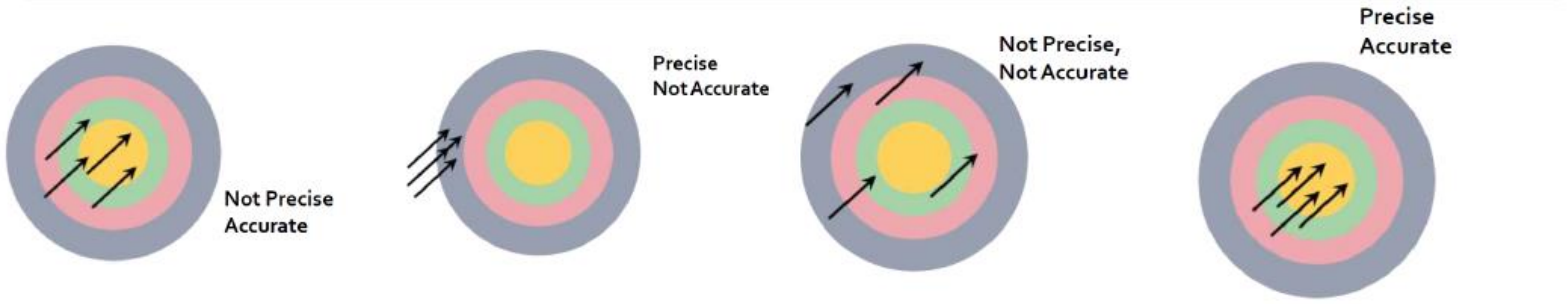
Accuracy and Precision:

1. Accuracy

Accuracy is defined as the degree of **closeness** of a measured value compared to the true value of the quantity to be measured.

2. Precision

Precision is defined as the degree of similarity of repeated measurements.



2.3 Measurements Characteristics:

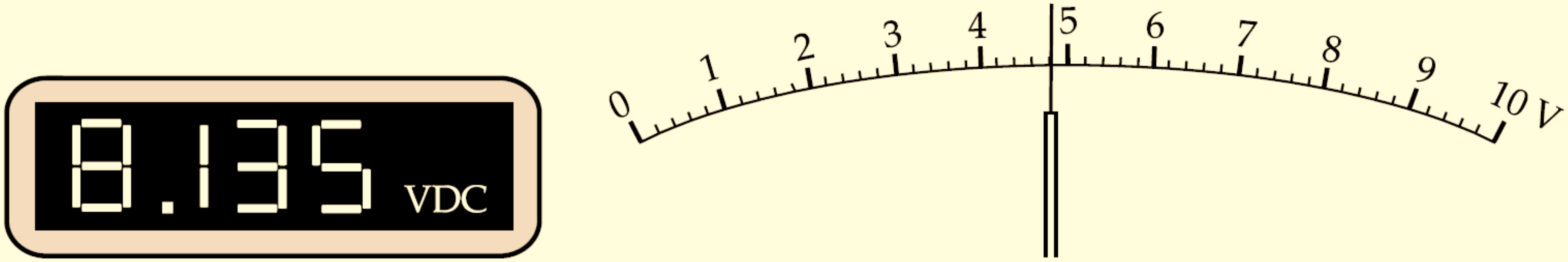
Resolution and Significant Figure:

3. Resolution

Resolution is defined as the **smallest change** in the measured quantity to which an instrument will respond.

4. Significant Figure

Significant Figure is defined as **the number of digits used** to represent a measured value. The more the number of significant figures, the more precise is the quantity.



(a) Digital voltmeter display with a 1 mV precision

(b) Analog instrument display with a 50 mV precision

Figure 2-4 Measurement precision depends on the smallest change that can be observed in the measured quantity. A 1 mV change will be indicated on the digital voltmeter display above when the quantity changes by ±0.001 V. For the analog instrument, ±50 mV is the smallest change that can be noted.

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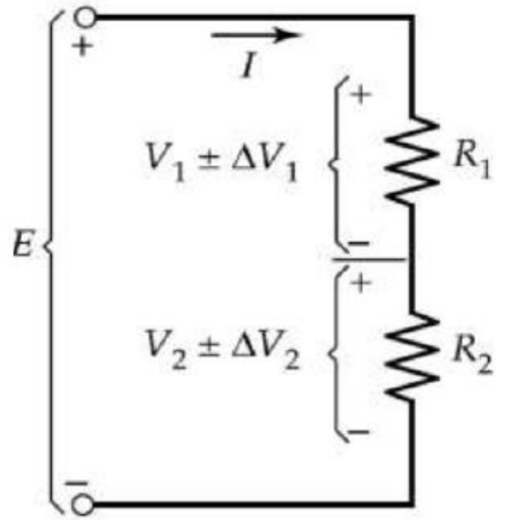
2.4 Measurement Error Combinations:

When a quantity is calculated from measurements made on two (or more) instruments , it must be assumed that the errors due to instrument inaccuracy combine in worst possible way. The resulting error is then larger than the error in any one instrument.

2.4 Measurement Error Combinations:

1. Errors in Sum of quantities

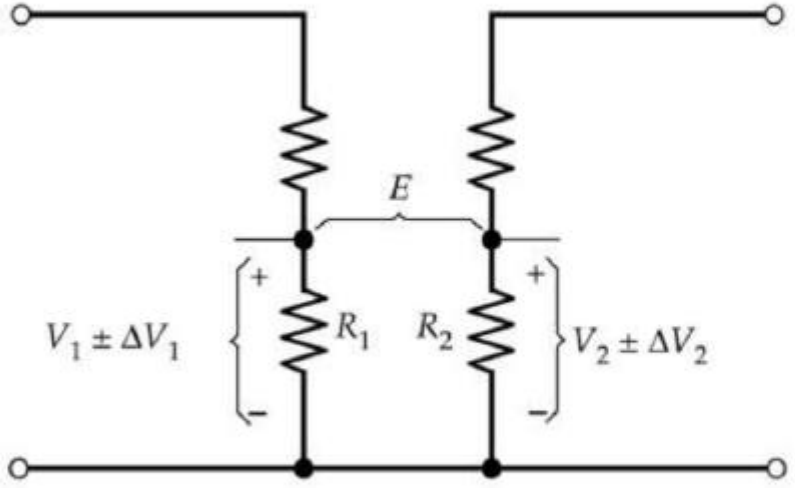


$$
\begin{aligned}
E &= V_1 + V_2 \\
&= (V_1 \pm \Delta V_1) + (V_2 \pm \Delta V_2) \\
&= (V_1 + V_2) \pm (\Delta V_1 + \Delta V_2)
\end{aligned}
$$

Error in Sum

Error in the sum of quantities equals the sum of absolute errors.

2. Errors in Difference of quantities



$$
\begin{aligned}
E &= V_1 - V_2 \\
&= (V_1 \pm \Delta V_1) - (V_2 \pm \Delta V_2) \\
&= (V_1 - V_2) \pm (\Delta V_1 + \Delta V_2)
\end{aligned}
$$

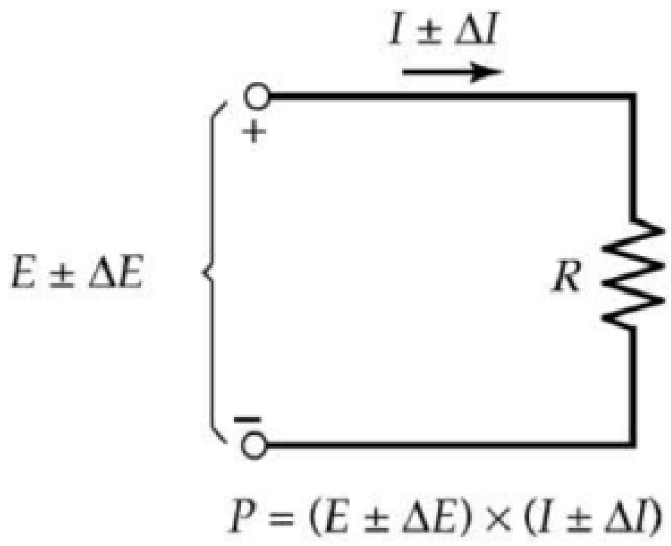
Error in Difference

Error in the difference of quantities equals the sum of absolute errors.

2.4 Measurement Error Combinations:

3. Errors in Product of quantities

$$P = EI = (E \pm \Delta E) \times (I \pm \Delta I)$$
$$= E.I \pm E.\Delta I \pm I.\Delta E \pm \Delta E.\Delta I$$
$$\approx E.I \pm E.\Delta I \pm I.\Delta E \quad (\Delta E.\Delta I \text{ is very small})$$



Percentage error in P is

$$\%P = \frac{E.\Delta I + I.\Delta E}{E.I} \times 100\%$$
$$= \left(\frac{\Delta I}{I} + \frac{\Delta E}{E}\right) \times 100\%$$
$$= (\% \text{ error in } I) + (\% \text{ error in } E)$$

Percentage error in the product of quantities equals the sum of percentage errors

2.4 Measurement Error Combinations

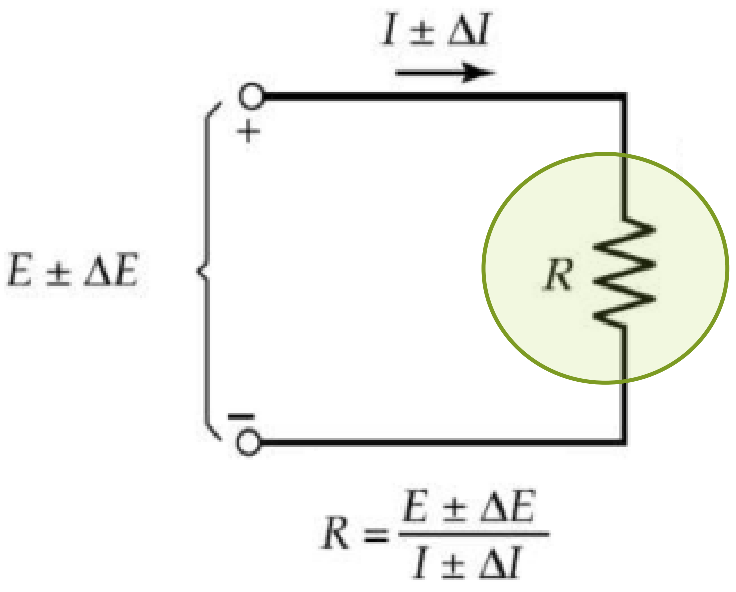
4. Errors in Quotient of quantities

$$R = \frac{E \pm \Delta E}{I \pm \Delta I}$$

Percentage error in R is

$$\%\text{error in } R = (\%\text{ error in } I) + (\%\text{ error in } E)$$

Percentage error in the quotient of quantities equals the sum of percentage errors



2.4 Measurement Error Combinations

5. Errors in quantity raised to a power:

$$P = I^2 R$$

Quantity raised to a power:

$$\%\text{error in } A^B = B(\%\text{ error in } A)$$

Summary

For $X = A \pm B$,	$error\ in\ X = \pm[(error\ in\ A) + (error\ in\ B)]$
For $X = AB$,	$\%\ error\ in\ X = \pm[(\%\ error\ in\ A) + (\%\ error\ in\ B)]$
For $X = A/B$,	$\%\ error\ in\ X = \pm[(\%\ error\ in\ A) + (\%\ error\ in\ B)]$
For $X = A^B$,	$\%\ error\ in\ X = \pm B(\%\ error\ in\ A)$

Example 1:

Calculate the maximum percentage error in the sum of two voltage measurements when $V_1 = 100\ \text{V} \pm 1\%$ and $V_2 = 80\ \text{V} \pm 5\%$.

Solution

$$V_1 = 100\ \text{V} \pm 1\%$$
$$= 100\ \text{V} \pm 1\ \text{V}$$

$$V_2 = 80\ \text{V} \pm 5\%$$
$$= 80\ \text{V} \pm 4\ \text{V}$$

$$E = V_1 + V_2$$
$$= (100\ \text{V} \pm 1\ \text{V}) + (80\ \text{V} \pm 4\ \text{V})$$
$$= 180\ \text{V} \pm (1\ \text{V} + 4\ \text{V})$$
$$= 180\ \text{V} \pm 5\ \text{V}$$
$$= 180\ \text{V} \pm 2.8\%$$

Example 2:

Calculate the maximum percentage error in the difference of two measured voltages when $V_1 = 100\ V \pm 1\%$ and $V_2 = 80\ V \pm 5\%$.

Solution

$$\left.\begin{aligned} V_1 &= 100\ V \pm 1\ V \\ \text{and} \quad V_2 &= 80\ V \pm 4\ V \end{aligned}\right\}$$

$$E = (100\ V \pm 1\ V) - (80\ V \pm 4\ V)$$

$$= 20\ V \pm 5\ V$$

$$= 20\ V \pm 25\%$$

Example 3:

An 820 Ω resistance with an accuracy of ± 10% carries a current of 10 mA. The current was measured by an analog ammeter on a 25 mA range with an accuracy of ±2% of full scale. Calculate the power dissipated in the resistor, and determine the accuracy of the result.

Solution

$$P = I^2 R$$

$$P = (10\ \text{mA})^2 \times 820\ \Omega$$

$$= 82\ \text{mW}$$

$$error\ in\ R = \pm 10\%$$

$$error\ in\ I = \pm 2\%\ \text{of}\ 25\ \text{mA}$$

$$= \pm 0.5\ \text{mA}$$

$$= \frac{\pm 0.5\ \text{mA}}{10\ \text{mA}} \times 100\%$$

$$= \pm 5\%$$

$$\%\ error\ in\ I^2 = 2(\pm 5\%)$$

$$= \pm 10\%$$

$$\%\ error\ in\ P = (\%\ \text{error in}\ I^2) + (\%\ \text{error in}\ R)$$

$$= \pm(10\% + 10\%)$$

$$= \pm 20\%$$

END OF LECTURE

BEST WISHES